The relationship between place value understanding, arithmetic performance and mathematics achievement in general

Basamak değeri kavramı anlayışı, matematik başarısı ve aritmetik performansı arasındaki ilişki

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Abstract. The purpose of this study was to investigate the relationship between the understanding of place value concept, arithmetic performances and mathematics achievement among fourth graders. A total of 250 students from schools in a mid-Anatolian town was participated in the study. A mathematics achievement test, arithmetic performance test and place value test were used as the data collection tools. The independent samples t-test, correlation coefficient and regression analysis were used in the analysis of the data. Results showed that there were no significant differences between the mathematics achievement, arithmetic performances and place value conceptions of boys and girls. A strong relationship appeared between place value conceptions, mathematics achievement and arithmetic performance of the students. An additional regression analysis showed the most important variable predicting the mathematics achievements of these students was the place value conception. 70% of the variance in mathematics achievements was explained by the place value conception scores. When the place value conception and the arithmetic performances of students are combined, they explained 77% of the mathematics achievement. Based on these results, it can be concluded that understanding place value is an important milestone in learning arithmetic and mathematics in general.

Keywords: Mathematics achievement, place value, arithmetic performance, primary student


Anahtar Sözcükler: Matematik başarısı, basamak değeri, aritmetik performansı, ilkokul öğrencileri
INTRODUCTION

Each digit in multi-digit numbers takes a value depending on its position. This is called place value (PV). The concept of PV constitutes the substructure of many topics in mathematical learning. Thorough understanding of this concept has a significant effect on learning many such topics in mathematics as whole numbers, decimal fractions, problem-solving, percentages and operations with fractional numbers (Andreasen, 2006; W. W. L. Chan, Au, & Tang, 2014). The understanding of the PV concept appears to be a building block especially in the development of multi-digit numbers and operations with them (Dietrich, Huber, Dackermann, Moeller, & Fischer, 2016). Thus, students' understanding of PV seems to be a prerequisite for the learning of further mathematics (McGuire & Kinzie, 2013; Nataraj & Thomas, 2007).

In many parts of the world, children are introduced to the PV concept when they start primary school. For example, students in Hong Kong learn two-digit numbers in the first grade, three and four-digit numbers in the second grade and five-digit numbers in the third grade. At the end of the third grade, they are required to know how the PV concept is applied to multi-digit numbers (W. W. L. Chan et al., 2014). In Turkey, children are introduced to the PV concept as finding tens and ones in numbers up to 20 only, in the first grade at primary school. Combining tens and ones to form a 2-digit number and identifying tens and ones in a number beyond 20 and up to 100 are introduced in the second grade. The PV concept is then strengthened by introducing three-digit numbers in the third grade and four, five, six-digit numbers in the fourth grade (Ministry of Education, 2015).

Understanding and efficiently using PV concept in mathematics is probably one of the greatest difficulties related to numbers that students encounter until that moment. Although students are expected to understand the PV concept by the second grade, the results obtained from studies indicate that a thorough understanding does not occur until much later (Cooper & Tomayko, 2011). The weak or fragile understanding of the PV concept may hinder students' further learning (Nataraj & Thomas, 2007). The difficulties or delays in learning PV concept might also be a prewarning for mathematical learning difficulties (B. M. Chan & Ho, 2010). The students who do not build the PV concept on solid bases, may be slower in acquiring and executing basic algorithms of arithmetic due to a lack of conceptual understanding (Cooper & Tomayko, 2011). In other words, a possible consequence of a delay experienced in meaningfully learning PV concept may cause misconceptions in primary and secondary school students, preventing the development of operations with multi-digit numbers (Byrge, Smith, & Mix, 2014).

The studies conducted on PV understanding are gathered into three categories. The first aims at determining the difficulties that students experience (Dinç-Artut & Tarım, 2006; Fuson, 1990; MacDonald, 2008; Thompson & Bramald, 2003; Tosun, 2011) and identifying students' mistakes (Tarım & Dinç-Artut, 2013; Tarım & Siyer, 2017) in relation to the PV concept. For example, a study conducted by Cooper and Tomayko (2011) revealed that children who did not have the idea of PV read the numbers “26” and “62” identically. Similarly, Thouless (2014) stated that students have difficulty in understanding the decimal base concept and this affects their ability to solve mathematical word problems correctly.

The students' restricted understanding of the PV concept is observed in studies carried out in Turkey as well. In a study conducted by Dinç-Artut and Tarım (2006), for example, when primary school students were asked to give as many counting stamps as the number in the units digit of “16”, they gave the correct number of objects (6 stamps), however, they gave one stamp instead of 10 stamps for the number in the tens digit. Similarly, another study conducted by Tosun (2011) found that some 5th graders still cannot distinguish that the place of a digit in a number affects its value. It indicates that these students have a restricted understanding of the PV concept.

A second category of studies are experimental studies aiming at overcoming students' difficulties in learning PV concept (Broadbent, 2004; Kamii & Joseph, 1988; Moore, 1992; Mutlu & Sarı, in press; Schmidt, 1995; Valeras & Becker, 1997). For example, it was observed that using concrete materials (decimal base-ten blocks, unit cubes, etc.) with “game-based activities” designed by Broadbent (2004) improved students' levels of understanding the structure of the
counting system paving the road to place value concept. Similarly, a study by Nataraj and Thomas (2007) aimed at developing the place value concept in different number bases by using concrete materials with the historical development of the number system. As a result of the study, it was observed that middle school students could develop the place value concept better by using concrete materials in a historical context.

In a third category of studies in the literature, it is observed that the relationship between the understanding of the PV concept and arithmetic performance is the focus (Dietrich et al., 2016; Ho & Cheng, 1997; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011). For example, in a study conducted by Moeller et al. (2011), it was revealed that building the place value concept in a successful way in the first grade is a reliable predictor of arithmetic skills in the third grade. Similarly, it was found that children with a poor understanding of the place value concept in primary school have poor results in addition and subtraction operations afterwards (Ho & Cheng, 1997).

The studies conducted on the PV concept are mostly descriptive, in other words, they usually focus on the difficulties experienced by students in relation to the PV concept (Arslan, Yildiz, & Yavuz, 2011; Dinç-Artut & Tarım, 2006; Tarım & Dinç-Artut, 2013; Tosun, 2011). Considering that the deficiencies in the understanding of place value at early ages may sustain their effect on the following complicated arithmetic processes (Moeller et al., 2011), this study aimed at examining the differential effects of PV understanding on fourth graders’ mathematics achievements and arithmetic performances. We also looked at the gender differences if any, since sometimes developmental differences are reported in boys and girls that may cause individual differences in mathematics achievement. The gender issue in education appears as the most popular study, especially in the international exams. The gender difference in mathematics of the countries participating in the international exams such as PISA, TIMSS and PIRLS varies. For example, when the results in mathematics literacy in the PISA 2015 are examined on the basis of all countries, it is observed that male students are more successful than female students (Eurydice, 2010). In Turkey, the difference between the average points of female and male students in the PISA 2015 is 6. However, this difference between females and males is not statistically significant for Turkey (Taş, Arıcı, Ozarkan, & Özgürlük, 2016). The gender difference in mathematics is not that clear and stable as in reading achievement (Eurydice, 2010). Since it has been discussed whether the gender factor has an effect on the place value concept in the previous studies in the literature, it is considered worthwhile to examine it in this study. Specifically, we looked at the relationships among place value understanding, mathematics achievements, and arithmetic performances.

**METHOD**

The relational survey design was used in this study, in which the relationships between the place value understanding, mathematics achievements, and arithmetic performances of primary school students were examined.

**Participants**

A total of 250 students were participated in the study, 119 girls, and 131 boys. The sample was drawn among 4th graders from four primary schools located in the central district of a mid-Anatolian city in Turkey. The convenience sampling method was used in the determination of the sample to be obtained from the population (Büyüköztürk, Çakmak, Akgün, Karadeniz, & Demirel, 2008). Schools were located in low to middle socioeconomic areas. There were totally 270 fourth graders in schools. All but 12 students, who were absent during data collection, were participated in the study.

**Data Collection Tools**

We have used three different tests to assess students’ performances relevant to the aim of the study; namely, a place value test (PVT), mathematics achievement test (MAT), and an
arithmetic performance test (APT). Data were collected by the researchers in schools to which the students were attending by conferring with their teachers for most appropriate time to students. The students and their teachers were informed about that the participation was voluntary.

The PVT was developed by the researchers. The test includes 17 questions prepared based on the Turkish fourth grade mathematics curriculum in primary school. The test was then sent to two mathematics educators and an expert in the field of assessment and evaluation, and they were asked to examine whether it complies with the intended outcomes in the curriculum. Three questions in the test were removed in accordance with the feedback provided by the experts. The final version of the PVT consists of 14 questions. It includes reading numbers, writing numbers, place value, face value (e.g. 13 represents 3 objects and the 1 represents 1 objects), grouping of numbers and regrouping of numbers. The test was piloted to 175 primary school 4th grade students. The KR-20 coefficient was calculated with the item difficulty and discrimination index using the data obtained. The average difficulty index of the test was calculated to be .68, and the KR-20 reliability coefficient was calculated to be .84. Considering these results, it is possible to say that the test is valid and reliable (Turgut, 1990).

The APT was developed by De Vos (1992) and adapted into Turkish by Olkun, Can, and Yeşilpınar (2013). It contains five columns of pure numerical arithmetic operations. It has 200 questions in total, 40 in each column, being addition in the first column, subtraction in the second column, multiplication in the third column, division in the fourth column and mixed operations in the fifth column from simple to more complex ones. It is suggested that the students were given one minute for each column. As suggested, Olkun et al. (2013) found a KR-20 reliability coefficient of .95 for the test implemented within time constrain. In the second application of the test students were allowed to do the test as much as they can within a class hour. In this implementation, the KR-20 coefficient was calculated to be .92.

The MAT was developed by Fidan (2013) according to the numbers domain of the Turkish mathematics curriculum for fourth graders (Education, 2015). It includes topics such as counting numbers, number patterns, and word problems including operations with whole numbers and fractions. The KR-20 reliability coefficient of the test was found to be .96. In the current study, the reliability coefficient of the test was calculated to be .93. The duration of the test is one class hour.

**Data Analysis**

Before the analysis, normality checks were carried out. Outliers, 8 students, were removed during the normality checks. Analyses were carried out on data from 250 students. The independent samples t-test was used in the data analysis to examine whether the total points obtained from the mathematics achievement test, arithmetic performance test and place value conception test vary by the gender factor. The Pearson product-moment correlation coefficient was calculated to reveal the relationship between the total points of the three tests used in the study. The multiple stepwise regression analysis technique was used to reveal the variables predicting mathematics achievement.

**FINDINGS**

We first conducted an independent samples t-test to reveal whether 4th-grade students’ PVT, MAT, and APT scores vary by the gender. There was virtually no difference between boys and girls in three of the tests used, for PVT $t(248) = -.290, p > .05$, MAT $t(248) = 1.424, p > .05$, and for APT $t(248) = .909, p > .05$. Therefore, we combined the data in terms of gender.

A multiple stepwise regression analysis was conducted to determine whether the students’ understanding of the place value concept and arithmetic performance scores are significant predictors of mathematics achievements. Before proceeding to the regression analysis, the correlation coefficients between the tests used in the study were examined. The results of the correlation coefficient calculation are presented in Table 1.
Table 1. The correlation result of the mean scores obtained from the PVT, MAT and APT

<table>
<thead>
<tr>
<th>Variables</th>
<th>Pearson Correlation</th>
<th>MAT</th>
<th>APT</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVT</td>
<td>Coefficients (r)</td>
<td>.839*</td>
<td>.813*</td>
</tr>
<tr>
<td></td>
<td>Significance (p)</td>
<td>.000</td>
<td></td>
</tr>
</tbody>
</table>

*p<.01

As seen in Table 1, there is a positive and significant correlation between the students’ PVT scores and MAT scores, (r=.839; p<.01). Similarly, a positive and significant correlation appeared between the students’ PVT scores and APT scores, (r=.813; p<.01). Considering the correlation coefficients, it is possible to say that the students’ understanding of the place value concept has a strong relation to both mathematics achievement and calculation performance.

The results of the multiple stepwise regression analysis conducted to determine the predictive power of the PVT and APT scores on student’s mathematics achievement test (MAT) scores are presented in Table 2.

Table 2. The results of the regression analysis for the determination of the variables affecting mathematics achievement

<table>
<thead>
<tr>
<th>Model</th>
<th>Variables</th>
<th>R</th>
<th>R²</th>
<th>F</th>
<th>β</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PVT</td>
<td>.839(a)</td>
<td>.704</td>
<td>588.60</td>
<td>.854</td>
<td>2.279</td>
<td>.000</td>
</tr>
<tr>
<td>2</td>
<td>PVT, APT</td>
<td>.879(b)</td>
<td>.773</td>
<td>420.97</td>
<td>-.549</td>
<td>-1.499</td>
<td>.000</td>
</tr>
</tbody>
</table>

a Predictors: (Constant), PVT (Place Value Test)
b Predictors: (Constant), PVT, APT (Place Value Test & Arithmetic Performance Test)

Considering the significant variables predicting the mathematics achievements of students (Table 2), it is observed that the place value conception (R= 0.839, R²= 0.704) was the most important variable in predicting mathematics achievement (F(1, 248)= 588.60, p<.01). In other words, the place value concept explains 70% of the student’s mathematics achievement. When the place value test scores and the students’ arithmetic performance test scores are included in the regression, these two variables (R= 0.879, R²= 0.773) explained 77% of variance in mathematics achievement (F(2, 247)= 420.97, p<.01).

DISCUSSION and CONCLUSION

The place value concept is one of the big and abstract ideas in elementary mathematics. Yet, it provides a basis for students’ understanding of further mathematical concepts such as operations with large numbers and fractions. It establishes connections between different mathematical ideas such as numbers, decimals, and measurement leading the development of mathematical skills for the formation of more advanced mathematical concepts (Kaplan, 2008). Considering the importance of the PV concept it seems worthwhile to investigate relationships between mathematics achievement, arithmetic performance and place value conception in this study.

First of all, there is no gender differences with respect to scores obtained from PVT, MAT, and APT. Therefore, we combined the data and no longer looked at the gender differences. Similar results are observed on place value and other related concepts in mathematics in the literature (Olkun, Mutlu, & Sarı, 2017; Olkun, Yıldız, Sarı, Uçar, & Turan, 2014; Tari & Dinç-Artut, 2013; Tosun, 2011). For many years, psychologists and mathematics educators have sustained their studies on the issue of whether gender has an effect on mathematics achievement. Although gender is an important variable causing personal differences in learning
environments, it does not create a significant difference among learners while learning mathematics (Hyde, Fennema, & Lamon, 1990; Lindberg, Hyde, Petersen, & Linn, 2010).

We found high levels of correlations among mathematics achievement, arithmetic performance and place value conceptions of fourth graders. As the place value scores of students increase, their mathematics achievements scores also increase. Similarly, as the place value scores of students increase, their achievements in arithmetic operations also increase. As a result of the regression analysis, it is observed that the most important variable predicting the mathematics achievements of fourth graders is the place value conception. Over 70% of the variance in students' mathematics achievement scores are explained by the place value conception. There are similar results in the literature. If the students established a firm background for the concept of place value they have more chance to achieve more in mathematics (Byrge et al., 2014; Dietrich et al., 2016; McGuire & Kinzie, 2013; Moeller et al., 2011; Nataraj & Thomas, 2007). Failure in understanding place value restricts the subsequent learning of mathematics (Byrge et al., 2014). It is expressed that the gaps occurring especially in the place value understanding of primary school students may maintain their effect on further arithmetic processes (Moeller et al., 2011). If this deficiency in understanding is not corrected, the gap resulting from the place value idea makes it difficult for children to cope with more complicated algorithms (Cuffel, 2009).

In conclusion, more emphasis should be placed on establishing stronger background for place value before advancing on operations with large numbers and fractions in order to support their long-term mathematical developments. Considering the fact that it is too late for coping with the place value concept in fifth or sixth grades (Byrge et al., 2014), it is highly important to meaningfully structure the place value concept starting from the earlier grades by paying special attention to the difficulties and misconceptions experienced by students while learning these concepts. Building the place value concept on a solid base, especially in students with low mathematics achievement is considered important in terms of providing convenience for their further mathematics achievements and learning many mathematical concepts such as decimals, fractions, percentages, comparison, and ordering, etc.). Further research may focus on developing meaningful contexts for learning place value concepts at early grades.

REFERENCES


