Exploring self-efficacy beliefs within the context of teaching mathematics with concrete models

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Abstract: The purpose of this study was to investigate pre-service mathematics teachers' self-efficacy beliefs about using concrete models in teaching mathematics. Participants were 31 junior pre-service middle school mathematics teachers. Six instructional sessions based on using concrete models were carried out during a three week period. Data were collected by a survey on pre-service teachers' efficacy beliefs about using concrete models and semi-structured interviews. Data analysis demonstrated that the instruction had positive contributions on the pre-service teachers' self-efficacy beliefs. In addition, results revealed that pre-service teachers had confidence in themselves about using concrete models both as learners and as teachers. Moreover, they believed that using concrete models would have positive consequences in teaching process and students' learning. However, pre-service teachers had relatively low efficacies about classroom management.

Keywords: Concrete models, pre-service mathematics teachers, self-efficacy beliefs, teachers' sense of efficacy

INTRODUCTION

As teachers develop experience with a novel curriculum material, they also develop a sense of efficacy for teaching by it. Accordingly, teachers' beliefs in their competencies about using a curriculum material play an important role concerning their performances. Therefore, the primary purpose of this study was to investigate pre-service mathematics teachers' self-efficacy beliefs on teaching mathematics with concrete models. The belief in one's capability to succeed in a particular situation is described as perceived self-efficacy by Bandura (1997) in his social learning theory. Similarly, Tschannen-Moran, Woolfolk Hoy and Hoy (1998) defined self-efficacy as “a future-oriented belief about the level of competence a person expects he or she will display in a given situation” (p.207). Self-efficacy beliefs affect how people feel, think, motivate themselves and behave (Bandura, 1994). In addition, they provide the foundation for human motivation, well-being, and personal accomplishment (Pajares, 2002). Moreover, these beliefs affect the choices individuals make because people engage in tasks in which they feel competent and confident and avoid those in which they do not feel so (Pajares, 1997, 2002). The other important concept in Bandura’s social learning theory is outcome expectancy that is distinct from perceived self-efficacy. Bandura (1997) argued that “perceived self-efficacy is a judgment of one’s ability to organize and execute given types of performances, whereas outcome expectancy is a judgment of the likely consequences such performance will produce” (p.21).

According to Tschannen-Moran et al. (1998), while the efficacy question is, “Do I have the ability to organize and execute the actions necessary to accomplish a specific task at a desired level?” (p.210), the outcome expectancy question is, “If I accomplish the task at that level, what are the likely consequences?” (p.210). Bandura (1997), on the other hand, suggested that there is a

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1 This article was produced from the master thesis of the first author.
causal relationship between beliefs of personal efficacy and outcome expectancies. In such a way the outcomes people expect depend mostly on their judgments of how well they will be able to perform in given situations.

Several studies indicate that teachers’ beliefs in their instructional efficacy partly determine how they structure academic activities in their classrooms (Bandura, 1997). Therefore, teacher efficacy has been an important field of research for years. According to Bandura (1997), teacher efficacy is a type of self-efficacy. However, it differs from self-efficacy in such a way that a person who has high self-efficacy about a specific task can still have a low sense of efficacy when it comes to teaching the task (Pajares, 1996). Tschannen-Moran et al. (1998) defined teacher efficacy as “the teacher’s belief in his or her capability to organize and execute [the] course of action required to successfully accomplish a specific teaching task in a particular context” (p.233). Alternatively, Wheatley (2005) described it as teachers’ belief about their ability to influence their students’ learnings.

In a study by Gibson and Dembo (1984), the researchers examined the relationship between teacher efficacy and observable teacher behaviors in which they suggested that teachers with high efficacy beliefs about teaching tend to devote more classroom time to academic activities, praise students’ academic accomplishment and work longer with difficult students. In contrast, teachers with low efficacy beliefs about teaching tend to spend more time on nonacademic activities, criticize students for their failures, and have lack of persistence in failure situations. Similarly, Bandura (1994) also argued that people with high sense of efficacy can resist the difficulties more than people with low efficacy beliefs. In this sense, both in-service and pre-service teachers’ self-efficacy beliefs are important areas for research focus. Moreover, besides their general teaching efficacy beliefs, their content specific efficacy beliefs have also been investigated in various research studies. This study is parallel to such research; in that, its main aim is to investigate a specific efficacy belief of pre-service teachers; that is their efficacy beliefs about concrete models. It is important to investigate future mathematics teachers’ self-efficacy beliefs about concrete models to understand the reasons for ineffective use of models, or worse, possible disuse. One important factor of teachers’ use of instructional strategies is their efficacy beliefs (Moyer, 2001). In this respect, as future practitioners, pre-service teachers are critical stakeholders whose self-efficacy beliefs need to be studied.

The concrete mathematical tools have been defined and named in different ways. While some researchers called them as manipulative or material, others called them as models. For instance, Moyer (2001), Karol (1991), Heddens (1997), and Uttal, Scudder and Deloache (1997) are the researchers who called the mathematical tools manipulatives or materials. Moyer (2001) defined concrete models as objects designed to represent abstract mathematical ideas clearly and concretely. She also added that they had both visual and physical attraction for learners. Similarly, Karol (1991) defined them as objects that students are able to see, feel, touch, rearrange and move. Both Moyer (2001) and Karol (1991) emphasized concrete models’ attraction for several senses of students. Otherwise, Heddens (1997) pointed out that they were objects from the real world which are used to show mathematical concepts. Besides, Uttal et al. (1997) suggested that they were designed specifically to help children learn mathematics. On the other hand, Van de Walle (2007) and Sowell (1989) called the mathematical tools concrete models. Van de Walle (2007) defined a concrete model as any object, picture, or drawing that is designed to represent abstract mathematical concepts. In addition, Sowell (1989) defined a concrete model learning environment as one where students work directly with models such as based-ten blocks, algebra tiles, geoboards, paper folding, or other concrete models under the supervision of a teacher. In the current study, the researchers refer to both mathematical tools and real life objects as “concrete models”, since there were activities with not only educational
materials (base-ten blocks, algebra tiles, pattern blocks, unit cubes, etc.) but also real life objects (water, glass, paper).

Considerable studies have supported the idea that using concrete models enhance learning of mathematics (e.g., Moyer, 2001; Silver, 2009; Sowell, 1989; Suydam & Higgins, 1977). The strongest theoretical arguments in favor of concrete models were developed by Piaget (1950), Bruner (1961), and Dienes (1967). Piaget believed that conceptual knowledge could not be transferred from one person to another; in contrast, he argued that it was developed by knower's own experiences (Steffe, 1990). Piaget (1950) also stressed the importance of concrete actions in learning mathematics. He stated that children, especially young ones, learn mathematics best from concrete activities. Therefore, he indicated that teachers could help students to develop more powerful ways of thinking by concrete activities. As a result, Piaget suggested that learning environments should include both concrete and symbolic models of the concepts. Similarly, Bruner (1961) provided additional evidence suggesting the need for firsthand student interaction with the environment. Moreover, in terms of concrete models, he tried to explain teacher's role and effective instruction by using them. According to Bruner (1961), "The devices themselves cannot dictate their purpose." (p.88). Therefore, the effectiveness of any technique or tool depends on teacher's skill and the instruction that is implemented. In a similar way, Dienes (1967) supported the use of concrete models and stressed the importance of learning mathematics by means of direct interaction with the environment. He believed that a child should recognize symbols as representations of concrete experiences before he or she uses the symbols in a mathematical system (Flener, 1980). In addition, like Piaget and Bruner, Dienes strongly suggested active student involvement in the learning process. In brief, each theoretician represented the cognitive viewpoints of learning and they suggested proper use of concrete models in mathematics classrooms (Post, 1981). Several studies support the idea that concrete models have positive effects on mathematics achievement of students of different age groups (Aburime, 2007; Bayram, 2004; Erşen, 2014; Kul, Çelik, & Aksu, 2018; Suydam, & Higgins, 1976). A study by Bayram (2004) investigated the effect of instruction with concrete models on eighth grade students' geometry achievement. A total of 106 eighth grade students participated in her quasi-experimental design. She found that students who received instruction with concrete models had higher scores on geometry achievement test than those who received instruction with traditional method. In short, she suggested that concrete models were beneficial for achievement of students. Similarly, Aburime (2007) investigated the effects of geometric models on mathematics achievement of high school students. The models in the study were eighteen different geometrical shapes constructed from cardboard paper. The sample for the study was 185 high school students. An experimental design was carried out. Experimental group students were taught with models while control group students were taught without models during ten weeks. The results suggested that students taught with geometric models had higher performance on mathematics achievement than students taught without models. In brief, like Bayram (2004), it was argued that geometric models had positive effect on achievement of students. Another study was conducted by Erşen (2014) with middle school students. She investigated the effects of mathematics teaching supported by materials on students’ achievement, attitudes and concerns. The selected mathematics subject was set theory. The sample for the study was 60 6th grade students. A quasi experimental design was carried out with an experimental group and a control group. The data analyses revealed a significant difference in mathematics achievement scores in favor of the experimental group. A review of research was conducted by Kul et al. (2018) in order to combine the empirical evidence regarding the effectiveness of educational materials in mathematics. The results of the meta-analysis of 54 studies revealed that using materials in mathematics has a positive and high influence on achievement. According to analysis of
mediator variables which are related to instructional characteristics, significant differences are found in the variables of mathematics topic, type of material, and application time.

However, there are also other studies which suggested that concrete models are not always necessarily more effective than traditional methods (e.g., Clements, 1999; Fennema, 1972; Van de Walle, 2007). Based on several studies, a reason for the ineffectiveness of concrete models is students' difficulty on achieving dual representations (Kaminski et al., 2006; McNeil, & Jarvin, 2007; Uttal et al., 1997). Students generally see models only as an object not a representation of a mathematical concept. Achieving dual representation means not only recognizing concrete model as a concrete object itself, but also as an abstract referent to a mathematical concept (Uttal et al., 1997). Realizing the underlying concepts of models, namely the relation between model and its intended referent is difficult for students (McNeil, & Jarvin, 2007; Uttal et al., 1997; Van de Walle, 2007). In this sense, Uttal et al. (1997) suggested that long term usage of models eases students’ understanding of models as representations of mathematical concepts and also increases their performance. Another common difficulty that students encounter is transferring their knowledge from a concrete environment to an abstract environment (Fuson, & Briars, 1990; Johnson, 1993; Kaminski et al., 2006; Uttal et al., 1997). Students fail to solve problems without models unless they are reminded to think about the models (Fuson, & Briars, 1990). For example, in the study by Uttal et al. (1997) a student could solve a problem such as 103+52 by using concrete model, but had difficulty in solving a written problem such as 12+14 without using the model although the second one was easier than the other. Because of this reason, Johnson (1993) recommended that a connection must be established in the activities that help the transition from concrete to abstract. In this sense, the main reason for possible ineffectiveness of models is quality of instruction. Therefore, teachers have an important role on the effectiveness of instruction with concrete models (Moyer, 2001; Post, 1981; Suydam & Higgins, 1977).

To date, different mathematical concepts have been taught to students of different age groups by using concrete models. In the USA, the Standards of National Council of Teachers of Mathematics (2000) recommend using mathematical concrete models at all grade levels. Similarly, the recent curriculum reform in Turkey emphasizes the use of concrete models in mathematics classrooms (Ministry of National Education, 2004; 2013; 2017). In addition, the new middle school mathematics curriculum in Turkey aims to facilitate students’ meaningful understanding of mathematics by using concrete models and other mathematical materials. In such a context, the role of teachers becomes critical, since they have an important function in the quality of mathematics instruction at the school level. In this sense, preparing pre-service teachers to meaningfully use concrete models in Turkish schools is an important issue. Moreover, research from several countries indicated that teachers’ usage of models is generally problematic (Moyer, 2001; Puncher, Taylor, O’Donnell, & Fick, 2008; Van de Walle, 2007). Therefore, it is critical to investigate future mathematics teachers’ self-efficacy beliefs about concrete models to understand the reasons for ineffective use of models, or worse, possible disuse. One important factor that contributes to teachers’ use of instructional strategies is their efficacy beliefs (Moyer, 2001). In this respect, as future practitioners, pre-service teachers are critical stakeholders whose self-efficacy beliefs need to be studied.

Considering the studies that investigate in-service mathematics teachers’ views about new instructional materials in Turkey, it can be concluded that teachers have generally positive views and they agree on the effectiveness of these materials on students' achievement, motivation and attitudes toward mathematics (Bal, 2008; Ersoy, 2005; Gurbuz, 2007). Similarly, pre-service teachers also have generally positive views about concrete models and want to use models in their future experience (Cakiroglu & Yildiz, 2007; Yetkin-Ozdemir, 2008). However, both pre-service and in-service teachers lack a clear idea about how models help students to understand mathematical concepts (Cakiroglu & Yildiz, 2007; Moyer, 2001; Yetkin-Ozdemir, 2008). They regard concrete models as motivating or reinforcing tools instead of tools to construct meaning (Cakiroglu & Yildiz, 2007; Howard, Perry, & Tracey, 1997; Moyer, 2001).
Therefore, they usually want to use models at the beginning of the lesson to introduce new concepts or after finishing an instructional unit to practice procedural skills or only for entertainment (Cakiroglu & Yildiz, 2007; McNeil & Jarvin, 2007; Moyer, 2001; Nevin, 1993).

To sum up, it can be concluded that teachers in Turkey generally have positive views about concrete models. However, they do not prefer to use them in mathematics classrooms, and their competencies about models are problematic. In addition, there are limited numbers of studies that investigate teachers’ competencies about concrete models. Thus, the main research questions of the study were, "What are the contributions of the instruction based on concrete models to the pre-service mathematics teachers’ self-efficacy beliefs and outcome expectancies about using concrete models in teaching mathematics?” and “What are the pre-service mathematics teachers’ self-efficacy beliefs and outcome expectancies about using concrete models after the instruction based on concrete models?”

**METHOD**

**Research Design and Procedure**

In this study, mainly one-group pretest-posttest research design was utilized. In this design, single group is measured twice; the first one before the treatment and the second one after the treatment (Fraenkel, & Wallen, 1996). Because this design does not have a control group, there is a serious limitation of ensuring that the change between the pretest and posttest is due to the treatment. To minimize such limitations of the design, quantitative findings were supported and mixed with qualitative methods. That is to say, a mixed method research that combines qualitative and quantitative approaches was utilized to investigate pre-service mathematics teachers’ self-efficacy beliefs (Creswell, 2007). For the quantitative part of the study, The Instrument of Pre-service Mathematics Teachers’ Efficacy Beliefs about Using Concrete Models (EUCM) developed by Bakkaloglu (2007) was administered both before and after the instruction. The treatment consisted of six instructional sections based on using concrete models in teaching mathematics was carried out during a three-week period. In order to get in-depth information about the pre-service teachers’ self-efficacy beliefs, two different data collection procedures, mainly questionnaires and interviews, were used. Creswell (2007) referred this type of data collection as ‘multiple source of information’. After the treatment, semi-structured interviews were conducted with 13 pre-service teachers during two weeks after the instruction. For each interview, first, the researcher explained the aim of the interview. Then, the students were asked questions prepared previously. After the pre-service teachers’ explanation, general inquiries were made, such as, “explain”, “clarify”, or “give details” and continued to ask more specific questions until a response was obtained. Interviews lasted approximately 45 minutes. This part of the study was based on qualitative research techniques that provide an in-depth explanation and rich description of the phenomenon (Creswell, 2007). The interview questions’ main aim was to collect data in order to answer the second research problem; and therefore, the questions prepared to get additional information on the pre-service teachers’ perceived self-efficacy beliefs about using concrete models and judgments about likely consequences of using them to teach mathematical concepts.

**Participants**

The participants of the first phase of the study were 31 junior pre-service middle school mathematics teachers enrolled in mathematics teaching program at a public university. There were 22 females and 9 males that took part in the study. The average age of the students was 21. For the second phase of the study, a total of 13 (9 females, 4 males) interviewees were selected from the participants of the experimental study regarding their self-efficacy gain scores that was the difference between pre and post-test self-efficacy scores. In order to select a group of participants that reflect a diverse range of opinions and self-efficacy beliefs about concrete models, they were selected among the pre-service teachers with highest and lowest gain scores. Firstly, participants were ordered according to their gain scores from high to low. Secondly,
they were divided into three groups in such a way that ten participants with highest gain scores were the first group, following eleven participants were the second group, and the last ten participants were the third group. Finally, 60% (6 out of 10) of the participants in the first group, 10% (1 out of 11) of the second group, and 60% (6 out of 10) of the third group were invited to participate to the interviews. All of these invited participants agreed to participate in the interviews. All of the interviewees had completed “Mathematics Teaching Method Course 1”. In addition, all of them were taking the “Mathematics Teaching Method Course 2” during the study. The aim of these method courses is to improve pre-service teachers’ mathematics teaching abilities, knowledge, beliefs and experiences. By the time of data collection, the interviewees had not taken any teaching practice course and they had not had any teaching experience with the concrete models.

**Data Collection Tools**

In the study, The Instrument of Pre-service Mathematics Teachers’ Efficacy Beliefs about Using Concrete Models (EUCM) developed by Bakkaloglu (2007) was used to measure the pre-service mathematics teachers’ efficacy beliefs about using concrete models in teaching mathematics. It was adapted from the Mathematics Teaching Efficacy Belief Instrument (MTEBI) developed by Enochs, Smith and Huinker (2000). EUCM had two factors consistent with previous instruments (Enochs, & Riggs, 1990; Enochs et al., 2000) and Bandura’s (1997) self-efficacy theory. These are personal efficacy beliefs about concrete model use (PECMU) and outcome expectancies regarding concrete model use (OECMU). EUCM consisted of 16 items; 10 items on PECMU subscale and 6 items on OECMU subscale. Items in the EUCM have a five-point Likert scale ranging from 1 to 5; 1 indicating ‘strongly disagree’ and 5 indicating ‘strongly agree’. Negatively worded items were reversed while scoring so that high scores on both subscales were the indicator of positive efficacy beliefs toward using concrete models in teaching mathematics. In addition, each mean score was calculated by dividing total scores by the number of participants.

In order to determine the internal consistency of the scale Cronbach alpha coefficient was used. Both the pre and post administration of the EUCM yielded Cronbach alpha coefficients of .74 for the subscale PECMU. In addition, for the subscale OECMU, pre and post administrations of the EUCM yielded Cronbach alpha coefficients of .7 and .8, respectively. Moreover, the Cronbach alpha coefficients for the total scale were .78 for pretest and .84 for posttest. Since the Cronbach alpha coefficient of a scale should be above .7 (Pallant, 2001), the Cronbach alpha coefficients were considered reasonable values for this study.

Interview was another important data collection tool because it enabled the researchers to investigate pre-service teachers’ efficacies in a more detailed way. The main aim of the questions was to get additional information on the pre-service teachers’ perceived self-efficacy beliefs and outcome expectancies about using concrete models to teach mathematical concepts. During the interview, general inquiries were made, such as, “explain”, “clarify”, or “give details”, and more specific questions were continued to ask until a response was obtained. Interviews lasted approximately 45 minutes. All the interviews were tape recorded and transcribed.

**The Process of Instruction**

During six sessions, instruction based on using concrete models in teaching mathematics, as a component of methods of mathematics teaching course was given to the junior pre-service mathematics teachers. It consisted of a variety of activities with the models. The activities were developed through a process of reviewing of resources from literature and the Ministry of National Education’ mathematics curriculum documents. The instructor started each session by distributing the models to the participants and then, she gave general information about the concrete models used in the session. Afterwards, activities with the models were carried out. Finally, the pre-service teachers had a discussion on the usage of the models in a real classroom. In Table 1, the used concrete models and the concepts that might be taught by using the concrete models are presented for each session.
Table 1. Concrete models used in instruction and corresponding concepts

<table>
<thead>
<tr>
<th>Sessions</th>
<th>Concrete Models</th>
<th>Concepts</th>
<th>Duration (minutes)</th>
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<tbody>
<tr>
<td>Session 1</td>
<td>Pattern blocks, Transparent fraction cards, Fraction bars, Triangular dot paper</td>
<td>Fractions</td>
<td>50</td>
</tr>
<tr>
<td>Session 2</td>
<td>Tens card, Hundreds card, Square paper, Based-ten blocks</td>
<td>Decimals, Percents</td>
<td>50</td>
</tr>
<tr>
<td>Session 3</td>
<td>Algebra tiles, Paper, Transparent counters, Glass</td>
<td>Algebraic expressions, Equations</td>
<td>50</td>
</tr>
<tr>
<td>Session 4</td>
<td>Hundred table, Transparent counters, Square dot paper, Based-ten blocks</td>
<td>Natural numbers, Integers, Square numbers</td>
<td>50</td>
</tr>
<tr>
<td>Session 5</td>
<td>Symmetry mirror, Geometry strips, Unit cubes, Square paper, Isometric dot paper</td>
<td>Two and three-dimensional shapes</td>
<td>50</td>
</tr>
<tr>
<td>Session 6</td>
<td>Squares set, Cubes set, Tangram, Square dot paper, Square geoboard, circular geoboard, Solid figures, Paper</td>
<td>Perimeters, areas and volumes of geometric shapes</td>
<td>50</td>
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</table>

The first session consisted of the concrete models that can be used for teaching fractions. These models were pattern blocks, transparent fraction cards, fraction bars, and triangular dot papers. To begin with, pre-service teachers were asked to model a fraction by using these concrete models. Then, the activities about equivalent fractions, comparing and ordering fractions, and operations with fractions were carried out by using the concrete models. The second session was about the concrete models that can be used for teaching decimals and percents. In this session, tens cards, hundreds cards, square papers and based-ten blocks were used to model decimals and percents, and activities with these models were carried out for the subjects of comparing and ordering decimals, and operations with decimals. In the third session, the concrete models that can be used for teaching algebraic expressions and equations were considered. At first, algebra tiles were introduced by giving details. Secondly, pre-service teachers were asked to model algebraic expressions and operations with these expressions. Thirdly, they were expected to factor algebraic expressions by using algebra tiles. Then, paper cutting activities were completed for modeling the identities. Finally, linear equations were solved by using transparent counters. In the fourth session, the hundred table was used for the concepts: divisibility, prime numbers and multiples of natural numbers. Afterwards, pre-service teachers were required to model operations with integers by using transparent counters. Finally, based-ten blocks and square dot papers were used to find out square numbers. In the fifth and sixth sessions, the concrete models used in the concept of geometry were considered. In the fifth session, firstly, the pre-service teachers were expected to obtain different two-dimensional shapes from an unordered polygon by using symmetry mirror. Secondly, pattern blocks and colored papers were used for the concepts in transformation geometry. Thirdly, by using geometry strips, the participants were asked to discover the relationships between not only a triangle’s edges but also a parallelogram and a quadrangle. Eventually, the pre-service teachers were expected to draw two-dimensional views (top, front, and sides) of the three-dimensional buildings. In addition, they were asked to construct three-dimensional buildings by using the unit cubes and draw these buildings on isometric dot paper. In the last session, there were activities about perimeter, area and volume. At first, the participants were asked to construct different shapes by using squares sets, cubes sets and tangrams and to calculate the areas and perimeters of these shapes. Moreover, they were expected to construct some polygons on geoboard and estimate the areas of these polygons. Secondly, pre-service teachers were supposed to construct a cube, a rectangular prism and a square prism with unit cubes and then, they were expected to discover the volumes of these three-dimensional shapes. Thirdly, by using solid figures, the participants were expected to discover the relationship between a square...
based pyramid and a rectangular prism, and also the relationship between a circular cone and a circular cylinder. Finally, paper folding and cutting activities were carried out to discover the area of a circle, the surface area and the volume of a sphere.

Data Analysis

The pre-service mathematics teachers’ self-efficacy beliefs about using concrete models in teaching mathematics were evaluated through self-efficacy’s personal efficacy and outcome expectancy dimensions. For the quantitative data, inferential analyses were carried to evaluate pre-service mathematics teachers’ self-efficacy beliefs before and after the instruction. Since data were collected from one group on two different occasions, paired-samples t-test was used to determine whether there was a significance mean difference among pre-test and post-test scores (Pallant, 2007).

For the analysis of data collected by the interviews, the interviews were tape recorded and transcribed. Content analysis was used to break the data into manageable units on the basis of the codes created. As an initial attempt, data were coded based on two themes derived from the literature; personal efficacy and outcome expectancy. Then, categories and subcategories for each theme were formed by using the recurring patterns. In order to establish trustworthiness and reduce bias, coding of the data was independently conducted by researchers and a second coder who was informed about the dimensions of self-efficacy and data analysis framework of the study.

RESULTS

Personal Efficacy Beliefs and Outcome Expectancies Before and After the Instruction

In this section, the findings of the analyses to answer the first main problem are presented. In order to find out the differences between pretest and posttest personal efficacy and outcome expectancy scores, data were analyzed by using paired-samples t-test at the .05 significance level. Paired-samples t-test has two assumptions which are the difference variable should be normally distributed and the difference scores should be independent of each other (Green, Salkind, & Akey, 2000). Before conducting the analyses, the assumptions were checked. Since the assumptions were met, the analyses were carried on.

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<tr>
<th></th>
<th>Personal Efficacy</th>
<th>Outcome Expectancy</th>
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<td></td>
<td>( \bar{x} )</td>
<td>N</td>
</tr>
<tr>
<td>Pretest</td>
<td>3.02</td>
<td>31</td>
</tr>
<tr>
<td>Posttest</td>
<td>3.82</td>
<td>31</td>
</tr>
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</table>

As seen in the Table 2, there was a statistically significant increase in personal efficacy scores from pre-test (\( M=3.02, SD=.46 \)) to post-test (\( M=3.82, SD=.38 \)), (\( p<.05 \)), \( t (30) = 8.80 \). The eta squared statistic was .72 that indicated a large effect size (Cohen, 1988). This means that 71% of the variance in the personal efficacy scores could be explained by the instruction based on concrete models.

Similarly, there was a statistically significant increase in outcome expectancy scores from pre-test (\( M=3.77, SD=.38 \)) to post-test (\( M=4.11, SD=.47 \)), (\( p<.05 \)), \( t (30) = 4.57 \), as shown in Table 2. The eta squared statistic was .41 that indicated a large effect size (Cohen, 1988). This means that 41% of the variance of the outcome expectancy scores could be explained by taking instruction based on concrete models.

In Depth Analysis of Interview Data

In this section, the findings of the analyses of the interview data to answer the second research problem are presented. According to the analysis of interview data, pre-service
mathematics teachers’ personal efficacy beliefs and outcome expectancies about using concrete models in teaching mathematics were classified under some major categories and subcategories. The participants’ views, which were coded under these categories, are explained elaborately and they are summarized for a clear explanation (See Figure 1 and Figure 2).

**Pre-service Mathematics Teachers’ Personal Efficacy Beliefs about Using Concrete Models**

The first main category of personal efficacy was efficacy about using concrete models as learners. According to interview results, almost all of the pre-service teachers believed that they had enough knowledge about using concrete models. Similarly, when they were asked to express their overall skills about using concrete models, most of them indicated that they had enough skills in using these models. However, when the same questions were asked to the interviewees specifically about each model, some of the interviewees indicated that they had difficulties in using some models. For instance, three of them said: "If I have to tell the truth, I cannot draw three-dimensional shapes on isometric dot paper because it is very confusing.,” "I want to use base-ten blocks; but I am confused which block I will call ones or which block I will call tens. It is really difficult because the values of blocks differ for integers and decimals.,” "I am concerned about the use of transparent fraction cards for multiplication and division because I did not exactly understand how they are used for these operations.”

As seen in the first quote above, some pre-service teachers pointed out that they had difficulty in using the concrete models requiring spatial thinking skills such as drawing three-dimensional shapes on isometric dot paper or forming these shapes with cubes. In addition, as explained in the second quote, some participants were concerned about confusing the values of blocks while using base-ten blocks for integers and decimals. As stated in the last quote, some of them specified that they had difficulty in using transparent fraction cards for multiplication and division. All of the interviewees stated that they did not use the models during their own education. Since they did not learn concepts by the models, they had doubts about using them. These doubts were about remembering the names and instructional uses of the models.

The second main category of personal efficacy was efficacy about using concrete models as teachers. It had two subcategories that were personal efficacy beliefs about teaching the mathematical concepts and about classroom management. Regarding the efficacy about teaching mathematical concepts through models, there were participants believing that they could effectively teach mathematical concepts by using concrete models. Some of them even claimed that they could better explain mathematical concepts by using the models. However, when the participants were asked to explain their opinions elaborately about teaching process with the models, majority of them expressed doubts about effectiveness of their instruction with concrete models. All of these interviewees stated that lack of experience in teaching with concrete models was the foremost reason for their doubts. One of them stated:

“...When a student only learns how to use algebra tiles or fraction bars at the end of the lesson, the student does not learn much because the main aim of the teacher should be teaching operations on fractions by using fraction bars or teaching multiplication with algebraic expressions by using algebra tiles. On the other hand, the main aim is not teaching how to use these models. However, I think it is difficult to achieve this. Thus, I need more experience in teaching with concrete models.”
The participant above put forward that he had doubts about using concrete models as an end not as a means to an end. Other participants indicated that they had doubts concerning distracting from the main objective of the instruction. Furthermore, they believed that they need more experience in order to gain confidence in teaching with the models. Regarding the efficacy about classroom management while teaching the mathematical concepts with concrete models, majority of the interviewees believed that management was the most important difficulty they might encounter. The participants had doubts about the instruction with concrete models in a crowded class and time constrains. In addition, availability of concrete models was another doubt of pre-service teachers.

Another doubt of pre-service teachers was about planning the lesson in which concrete models were used. They did not have any clear idea about when and how they use the models during a lesson.

**Pre-service Mathematics Teachers’ Outcome Expectancies about Using Concrete Models**

The first main category was pre-service mathematics teachers' outcome expectancies regarding students' learning. It consists of three subcategories: outcome expectancies about cognitive learning, affective learning and psychomotor learning of students.
Under the cognitive learning category, participants seemed to be concerned about two major types of student outcomes: understanding of concepts and thinking skills. The first outcome expectancy about cognitive learning was about the influences of teaching mathematical concepts by using the models on students’ understandings of these concepts. Both the interview and posttest data indicated that pre-service teachers generally had positive expectancies about the likely consequences of the models on students’ cognitive learning. All interviewees pointed out that using concrete models enabled students to better understand the mathematical concepts. One of them stated:

“... If I teach equivalent fractions by using fraction bars, for example, let’s consider 1/2 and 2/4, students can easily see the equivalence between 1/2 and 2 pieces of 1/4. Generally, students cannot imagine it in their minds, the models make it concrete and it becomes easier to learn.”

As seen in the above quote, the participant indicated advantages of using concrete models for students’ understanding of concepts. Similarly, the participants stated that the models prevent memorization, increase retention of the concepts, enable students to establish connections among concepts, and make concepts more concrete. In addition, although majority of the pre-service teachers thought that the models increase achievement of students in mathematics, few interviewees stated that the models would not have any influence on students’ achievement. A participant said:

“If I have to tell the truth, I do not believe that concrete models increase the achievement of students all the time because the effect of the models highly depends on teachers. For example, if a teacher uses the models in a wrong way such as only for demonstrating, there will not be any success.”

This participant was aware of the limitations of the models emerging from the teacher. In addition, some pre-service teacher put forward other limitations of the models. For example, one of them stated: “Even though we use the concrete models, children still have to memorize certain things. At this time, they should memorize the rules of the models because some models really require memorization for proper use.” Like the interviewee in the quote, some pre-service teachers thought that several models require memorization. In addition, some of them believed that concrete models may prevent abstraction. Moreover, some interviewees pointed out that some concrete models may lead to confusions and even to misconception in students’ minds.

The second outcome expectancy about cognitive learning was influences of teaching mathematical concepts by using the models on students’ thinking skills. Majority of the pre-service teachers claimed that concrete models would improve logical thinking skills of students. In addition, most of the interviewees believed that concrete models developed students’ spatial thinking and creative thinking skills. According to the participants, especially geometric models such as tangram, unit cubes, cube sets or square sets developed students’ spatial and creative thinking skills.

The second subcategory of students’ learning was pre-service teachers’ outcome expectancies about students’ affective learning. It consisted of the likely consequences of teaching concepts by using concrete models on students’ attitude and motivation. Majority of the interviewees claimed that using concrete models in mathematics lessons enabled students to develop positive attitudes toward mathematics. They explained the reasons in various ways. For instance: “…because the models enable students to better understand mathematical concepts.”, “…because students do not need to memorize.”, “…because the models increase students’ achievements.”, “…because the models make mathematics an enjoyable course instead of a frightening course.” As seen in the first three quotes above, some of the reasons they envision for attitudinal impact were parallel with their expectancies about cognitive learning. In other words, the pre-service teachers believed that if students understand the concepts clearly and complete the tasks successfully, they develop positive attitudes toward mathematics. In addition, as indicated in the last quote, the interviewees believed that the models enabled students to enjoy...
the lesson, and therefore, students developed positive attitudes toward mathematics. Furthermore, some pre-service teachers indicated that concrete models enable students to develop positive attitudes not only toward mathematics, but also toward the teacher. In addition, nearly all of the interviewees believed that using concrete models in mathematics lessons increases students’ motivation, attract students’ attention and increase their willingness to attend the lesson.

**FIGURE 2.** A summary for the pre-service mathematics teachers’ outcome expectancies about using concrete models

The last subcategory of students’ learning was pre-service teachers’ outcome expectancies about students’ psychomotor learning. Majority of the participants claimed that concrete models positively affect students’ psychomotor learning. The pre-service teachers believed that activities with concrete models that required drawing, cutting, or combining pieces developed students’ psychomotor skills, and therefore, they positively affected their psychomotor learning.

The second main category of outcome expectancy was pre-service mathematics teachers’ outcome expectancies regarding teaching process. It consisted of outcome expectancies about teaching mathematical concepts and classroom management. The pre-service teachers believed that concrete models help teachers to teach mathematical concepts in an effective way. To
illustrate, an interviewee reported:

“... when I say 1/2, children can imagine it in their minds because it is half, thus it is easy to teach. But if I say 1/6 or 4/5, children cannot imagine these fractions in their minds. However, if I use fraction bars or pattern blocks, I can easily show 1/6 or else, I can show the fractions’ relationships with the whole, and therefore, children can simply imagine them. In short, the models make my job easier.”

This interviewee indicated that concrete models facilitated the process of teaching concepts. Some of them explained their claim by giving more details. For instance, they believed that concrete models facilitated the representation of mathematical concepts, figures and properties. In addition, some interviewees suggested that concrete models help students to answer questions in their minds such as why, how, etc. in an easy way. For instance, one of them stated:

“... by using geometry strips, students clearly see that in a triangle the addition of the lengths of 2 edges cannot be longer than the length of the 3rd edge, and the subtraction of the lengths of 2 edges cannot be shorter than the length of the 3rd edge. If we teach this to students as a rule, they may ask why it is true.”

The second subcategory of outcome expectancy was pre-service mathematics teachers’ outcome expectancies regarding classroom management. It was referred to as pre-service mathematics teachers’ judgments about likely consequences of using concrete models on classroom management. It consisted of pre-service teachers’ outcome expectancies regarding time, students’ reactions, safety of concrete models and noise.

Pre-service teachers believed that concrete models may cause some management problems. Some of them explained their claim by giving more details. For example, a participant stated: “... It is a great deal of time consumption, because all of the students in the class will try to make the activities with the models. Moreover, I should wait for all of them and check their work.”

Like the pre-service teacher in this quote, majority of the participants indicated that concrete models were time consuming. Furthermore, most of the participants believed that the students might regard the models as toys instead of mathematical tools. In addition, few participants specified that students might damage or lose models. Lastly, some interviewees indicated doubts about noise in the classroom. One of the participants stated: “Since most of the models are used in group work activities, students might speak with each other. Therefore, there might be so much noise in the classroom.”

Pre-service teachers indicated only one positive outcome expectancy regarding management. It was that concrete models increased students’ involvement in the lesson. For instance, one of them said: “Teaching concepts by using concrete models enables all students to attend the lesson because all of them must be involved in the activities, and they also must share their opinions with the whole class.” As indicated in this quote, pre-service teachers suggested that in a learning environment where concrete models are used, students are at the center of the lesson; in other words, they do the activities and find the rules by themselves, and share their ideas with each other. Therefore, the participants believed that concrete models increased students’ involvement in class activities.

**DISCUSSION and CONCLUSION**

Since the current study was a mixed method research that combines qualitative and quantitative approaches, both qualitative and quantitative results were used to get conclusions about pre-service mathematics teachers’ personal efficacy beliefs and outcome expectancies about using concrete models in teaching mathematics. The quantitative and qualitative analyses suggested that the instruction based on concrete models had positive contributions to the pre-service mathematics teachers’ self-efficacy beliefs about using concrete models. To illustrate, the paired-samples t-test between pretest and posttest personal efficacy and outcome expectancy scores concluded that there was a statistically significant increase in scores from
pre-test to post-test. In addition, in the interview, pre-service teachers indicated high levels of personal efficacy and outcome expectancy about using concrete models. The reason for the increase in pre-service teachers’ self-efficacies may be that the instruction enabled them to have knowledge and skills about most of the models. In addition, not only they learned how the models can be used for teaching mathematical concepts, but also they used the models as learners. Therefore, they had chance to examine their competencies about using the models and to determine the models’ likely consequences in a more objective way.

According to both quantitative and qualitative results of the study, pre-service teachers had confidence in their performances to be effective in using concrete models as learners. Yet, in the interview, when pre-service teachers were asked to explain their thoughts in a more detailed way, they indicated some difficulties in using several models. In addition, they maintained some doubts about using the models such as forgetting the names of the models or how to use them.

The difficulties and doubts that the pre-service teachers had may be due to the limited experience in using the models as learners. In fact, interview results suggested that all of the interviewees did not use the models during their own education. Although the pre-service teachers used the models in the instruction as learners, there was not enough time for being competent users of the models. Therefore, in undergraduate education, especially in mathematics teaching method courses, pre-service teachers should be given the chance of using the models as learners. Similarly, Cakiroglu and Yildiz (2007) underlined the importance of mathematics teaching method courses in undergraduate education. The results in the current study found that pre-service teachers had confidence about the effectiveness of their instruction with concrete models. However, teachers might declare that they feel confident even though they do not really feel confident at all (Wheatley, 2005). For this reason, in the interview, pre-service teachers were asked to express their ideas about themselves by giving more details. For instance, when they were asked to explain their judgments elaborately about teaching with the models, some of their doubts were revealed. According to the pre-service teachers, the foremost reason for their doubts was their lack of experience about teaching with the models. This might be a reason because pre-service teachers really have limited experience about teaching with models. In mathematics teaching method courses, pre-service teachers are asked to prepare a short lesson (usually 20 minutes) by using the concrete models for a small group of learners who are their peers. However, it is very different from teaching concepts to students in a real classroom environment. Moreover, in the teaching practice, they may not find enough opportunities to teach mathematics by using the models. In short, each pre-service teacher has a chance of using only limited numbers of models as a teacher. Therefore, in the mathematics teaching method courses and teaching practice, they should be provided more opportunities to practice teaching with models.

Another conclusion of the study was that there was an inconsistency with quantitative and qualitative results about pre-service teachers’ personal efficacy about classroom management. In the post-test, they had high efficacy beliefs about classroom management whereas in the interview they demonstrated low level of efficacy about it. Furthermore, in the interview, they stated that management was the most important difficulty that they might encounter while teaching the mathematical concepts by using the models. The reason for the inconsistency may be that pre-service teachers had confidence in general classroom management. However, they had some doubts about specific management problems such as time constrains, class size, availability of concrete models, and planning the lesson. In the interview, they had a chance to explain their doubts about these specific management problems; yet, in the post-test they had to think about the general situation. The specific management problems that pre-service teachers mentioned were similar to the views of pre-service and in-service teachers about using concrete models in the literature. For instance, Cakiroglu and Yildiz (2007) found that time constrains and availability of the models were two of the factors affecting pre-service teachers’ decision on whether or not to use models in teaching mathematics. Similarly, the in-service teachers in Ersoy’s (2005) study complained about insufficiency of instructional tools in mathematics classrooms. Moreover, the in-service...
teachers in both Ersoy's (2005) and Bal's (2008) studies complained about implementation problems of models due to crowded classes.

Both quantitative and qualitative results showed that the pre-service teachers had generally positive expectancies about the likely consequences of the models on students' learning. The results concluded that, in general, pre-service teachers believed that using concrete models positively affected students' learning. On the other hand, they were aware of some negative influences of the models. While they expressed only positive expectancies about affective and psychomotor learning, they indicated both positive and negative expectancies about cognitive learning.

As aforementioned, pre-service teachers not only indicated the advantages of the models on students' learning, but also stated some limitations of them such as preventing abstraction, requiring memorization, and leading to misconception. Although the participants did not receive an explicit instruction about such limitations, they seemed to develop ideas about possible weaknesses about the materials they had been learning. These limitations were similar to the limitations stated in the literature. For example, Uttal et al. (1997) found that students generally succeeded in solving problems by using concrete models, but they could not transfer their mathematical knowledge that was learned by models to an abstract environment. Johnson (1993) also recommended that a connection must be established in the activities that help the transition from concrete to abstract. In addition, Moyer (2001), and Szendrei (1996) indicated that some concrete models required memorization for proper use. Having critical thoughts about concrete models and developing awareness should be interpreted as a positive aspect of pre-service teachers' development. Especially, considering that the participants' critiques to the models demonstrated a similarity to the ones raised in the literature, we can see that they had been through an intense thinking process about the concrete models. In this sense such explicit training about concrete models are likely to trigger pre-service teachers' thinking process and help them to develop a critical perspective.

According to both quantitative and qualitative results of the study, pre-service teachers generally had negative expectancies about the likely consequences of the models on classroom management. For instance, they thought that using concrete models would be time consumption and cause noise in the class. As mentioned earlier, time constrain was also one of their difficulties in using the models as teachers. It can be concluded that when considering time, not only their personal efficacy beliefs were low, but also their outcome expectancies were negative. This might cause that they would not prefer to use concrete models in their classrooms. In addition, pre-service teachers believed that the students might regard the models as toys instead of mathematical tools. This may be explained by some of the limitations arising from concrete models themselves that were mentioned in the literature. For instance, McNeil and Jarvin (2007), and Uttal et al. (1997) argued that models with colorful and attractive design or familiar to students in outside of school contexts -such as toys- may lead students to see the activity as a game and see the models as toys. Furthermore, pre-service teachers indicated a physical limitation of the models that was lack of durability. It can be concluded that they were aware of some limitations arising from the characteristics of the concrete models themselves. However, the pre-service teachers were not so much aware of the limitations arising from the teacher. They mostly indicated the limitations arising from students or concrete models themselves. On the other hand, although pre-service teachers had mostly negative expectancies about classroom management, they believed that the models increased students' involvement in the lesson. Similarly, Karol (1991) and Nevin (1993) put forward that concrete models are to make students active participants in their own learning process.

To conclude, for increasing both pre-service and in-service teachers' self-efficacy beliefs about using concrete models in teaching mathematics, they should be taught the concrete models' both strengths and limitations, and also they should be given the opportunities to practice using concrete models both as learners and as teachers. In order to achieve the intended changes in pre-service mathematics teachers' self-efficacy beliefs about using concrete models, their personal efficacies and outcome expectancies on the subject of concrete
models should continue to be analyzed well. Moreover, further studies should be conducted not only with the pre-service mathematics teachers, but also with the in-service mathematics teachers. Besides, further research need to be done to explore how in-service mathematics teachers’ efficacies about using concrete models affect students’ learning in various topics. In addition, the continuum of development process of self-efficacy beliefs of pre-service and in-service teachers’ beginning from early stages of teacher training program and during their classroom practices should be examined and also the influence of experience or other related factors on the self-efficacy construct should be investigated.

REFERENCES


